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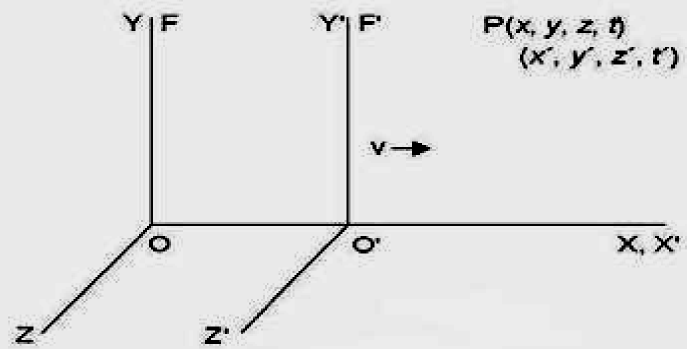
Lorentz Transformation

The invariance of speed of light in all inertial frames implies that Galilean transformation equations are not suitable. Therefore, we have to introduce new transformation equations which are consistent with the new concept of the invariance of speed of light in free space. These transformation equations were derived by Lorentz and are known as *Lorentz transformation equations*.

Let us consider two inertial frames F and F′ Again consider two observers O and O′ situated at the origin in the frame F and F′, respectively. Two coordinate systems coincide initially at $t = t' = 0$. Let a pulse of light be generated at time $t = 0$ from the origin which spreads out in the space and at the same time the frame F′ starts moving with constant velocity v along +ve direction of X-axis relative to the frame F. This pulse reaches at point P, whose coordinates of position and time are (x, y, z, t) and (x', y', z', t') measured by the observer O and O′, respectively.



Therefore, the transformation equations of x and x' can be written as,

$$x' = k(x - vt) \qquad \text{(i)}$$

where, k is a constant of proportionality and is independent of x and t.

The inverse relation can be written as,

$$x' = k(x' + vt') \qquad \text{(ii)}$$

Keeping in mind that the time t and t' are not equal, we put the value of x' from Eq. (i) in Eq. (ii), in order to get

$$x = k[k(x - vt) + vt']$$

$$\text{or} \quad \frac{x}{k} = kx - kvt + vt'$$

$$\text{or} \quad t' = \frac{x}{kv} - \frac{kx}{v} + kt$$

$$\therefore \quad t' = kt - \frac{kx}{v}\left(1 - \frac{1}{k^2}\right) \qquad \text{(iii)}$$

Now according to second postulate of special theory of relativity speed of light c remains constant. So the velocity of pulse of light which spreads out from the common origin observed by observers O and O′ should be the same. Therefore,

$$\left.\begin{aligned} x &= ct \\ x' &= ct' \end{aligned}\right\} \qquad \text{(iv)}$$

By putting the values of x and x' from Eq. (iv) in Eqs. (i) and (ii), we have,

$$ct' = k(x - vt) = k(ct - vt) \qquad \text{(v)}$$

or $$ct' = kt(c - v)$$

and $$ct = k(ct' + vt')$$

or $$ct = kt'(c + v) \qquad \text{(vi)}$$

By multiplying Eqs. (v) and (vi), we get

$$c^2tt' = k^2tt'(c^2 - v^2)$$

$$k^2 = \frac{c^2}{(c^2 - v^2)}$$

or $$k = \pm\frac{1}{\sqrt{1 - v^2/c^2}} \qquad \text{(vii)}$$

or $$\frac{1}{k^2} = 1 - \frac{v^2}{c^2}$$

or $$1 - \frac{1}{k^2} = \frac{v^2}{c^2} \qquad \text{(viii)}$$

Using Eqs. (i), (iii), (vii) and (viii), we have

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \qquad \text{(ix)}$$

$$t' = kt - \frac{kx}{v}\left(\frac{v^2}{c^2}\right)$$

$$= kt - \frac{kxv}{c^2} = k\left(t - \frac{xv}{c^2}\right)$$

or $$t' = \frac{\left(t - \frac{xv}{c^2}\right)}{\sqrt{1 - v^2/c^2}} \qquad \text{(x)}$$

$$y' = y \text{ and } z' = z \qquad \text{(xi)}$$

Hence, the transformation equations become

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}},\ y' = y,\ z' = z \quad \text{and} \quad t' = \frac{\left(t - \frac{xv}{c^2}\right)}{\sqrt{1 - v^2/c^2}}$$

Imagine if the frame F is moving with velocity v along the $-$ve direction of X-axis relative to frame F′, then we get transformation equations of the form

$$x = \frac{x' + vt'}{\sqrt{1 - v^2 / c^2}}, y = y', z = z' \quad \text{and} \quad t = \frac{\left(t' + \frac{vx'}{c^2}\right)}{\sqrt{1 - v^2 / c^2}}$$

These equations are known as *inverse Lorentz transformation* equations.

If the speed of moving frame is much smaller than the velocity of light c, (i.e., $v \ll c$) then the Lorentz transformation equations reduce to Galilean transformation equations.